Gravitational Many-Body Problem

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Talk structure

- 1. Overview of Gravitational Many-Body Problem
 - collisionless and collisional systems
 - Evolution of collisional systems
- 2. Some resent topics
 - Black hole formation in star clusters
- 3. Simulation methods

Gravitational Many-Body Problem

- Definition
- Astronomical examples
- Large-N limit "collisionless" systems
- Two-body (collisional) relaxation
- Evolution of collisional systems

Definition

$$m_i rac{d^2 x_i}{dt^2} = \sum\limits_{j
eq i} f_{ij}$$

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 x_i, m_i : position and mass of particle i f_{ij} : force from particle j to particle iGravitational (Newtonian) force:

$$f_{ij}=Gm_im_jrac{x_j-x_i}{|x_j-x_i|^3},$$

G: Gravitational constant

Astronomical Examples

Star Clusters



Galaxy



Galaxy Group



Clusters of Galaxies



Galaxy Cluster 0024+1645

Yellow: Galaxies in the cluster

Blue: background galaxies (enlarged and stretched through gravitational lens effect)

Large Scale Structure



SDSS Survey Radius: about 1Gpc (1/3 of the Universe)

Constituent of the Universe



70%: Interacts with nothing25%: Interacts only through gravity

Behavior of Dark Matter

Initial conditions

- Power spectrum of density fluctuation
- Cosmological parameters

Fairly well understood (if our assumption on the nature of DM is correct) We can simulate the structure formation through

- dark matter dynamics
- (Animation)

Large-N Limit — collisionless systems

Collisionless Boltzmann Equation (CBE)

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} f - \boldsymbol{\nabla} \Phi \cdot \frac{\partial f}{\partial \boldsymbol{v}} = \boldsymbol{0}, \qquad (3)$$

f: Distribution function in 6-dimensional phase space

 Φ : Gravitational potential, given by:

$$\nabla^2 \Phi = 4\pi G \rho. \tag{4}$$

 ρ : mass density

$$\boldsymbol{\rho} = \boldsymbol{m} \int \boldsymbol{d} \boldsymbol{v} \boldsymbol{f}, \tag{5}$$

Dynamical Equilibrium

Dynamical Equilibrium: Stationary solution of CBE+Poisson Eq.

Star clusters, Galaxies: Roughly in DE Large Scale Structure: NOT in DE (Dynamically too young)

Jeans' Theorem

Stationary solution of CBE in a given potential ϕ can be expressed as

$$f(x,v) = f[I_1(x,v), I_2(x,v), ...],$$
 (6)

where $I_1, I_2, ...$ are integrals of the motion.

Example: Spherically symmetric system: f = f(E, J). (energy and total angular momentum)

The distribution function f for a stellar system with finite mass cannot be Maxwellian, because fmust be zero for E > 0.

Dynamical and Thermal Equilibrium

Dynamical Equilibrium: Stationary state of CBE

No entropy production (no collision term)

 \rightarrow Not in thermal equilibrium Not in LOCAL thermal equilibrium ether (Jeans' Theorem)

Collision term

Two-body relaxation

Close encounter between two point-mass particles Orbit changes: similar to physical collision Relaxation timescale:

$$t_r \sim rac{v^3}{Gnm^2\log\Lambda}$$

 $\log \Lambda \colon$ Coulomb Logarithm, $\log \Lambda \sim \log N$

$$t_r \sim \frac{N}{\log N} t_d \tag{8}$$

 t_d : Dynamical timescale Relaxation timescale \propto Number of particles

Thermal (two-body) relaxation

- Important for
 - Small-N systems
 - Systems with small dynamical timescale
- Examples
 - Star clusters
 - Central region of galaxies
 - Clusters of Galaxies
 - Few-body systems (triple etc)

Virial Theorem and negative specific heat

For a stellar system in dynamical equilibrium

$$\boldsymbol{E} = -\boldsymbol{K} = \boldsymbol{V}/\boldsymbol{2} \quad (9)$$

- E: Total energy (=K+V)
- K: Kinetic energy
- V: Potential energy
- If a system loses energy $(\Delta E < 0)$, it becomes hotter $(\Delta K > 0)$



Gravothermal Instability

- Isothermal gas in a spherical adiabatic wall
- Radius: R
- Mass: M
- Central density: ho_0
- Density at the wall: ho_w
- $D=
 ho_0/
 ho_w$
- D=1: No gravity $D=\infty$: Singular solution $ho\propto r^{-2}$



Gravothermal Instability(cont'd)

For large D (D > 709):

move heat from the central region to outer Isothermal gas \downarrow central region becomes hotter due to negative specific hear \downarrow heat flow grows



Evolution in finite amplitude

- Numerical calculation Hachisu *et al.* (1978) : Gas dynamics
- Lynden-Bell & Eggleton (1980): Self-similar solution
- Cohn (1980): Direct integration of orbit-averaged Fokker-Planck equation



Energy Production

Gravitational contruction of star: halted by nuclear fusion

Stellar system: Three-body binary formation

Close encounter of three particles \rightarrow One binary + one single star: Statistically most likely outcome

Animation

Three particles

Gravothermal Oscillation



Sugimoto and Bettwieser (1983)

Three curves: Different energy production coefficients (Different N)

N-body simulation

1996: Confirmed with N-body simulation



Some recent topics

• Collision of stars and blackhole formation

IMBH candidate in M82

Matsumoto et al. ApJL 547, L25



X-ray sources. Brightest one: Corresponds to 1000-solar-mass black hole (Intermidiate-mass BH)

Subaru observation



Host star cluster

McCrady et al. 2003 (astro-ph/0306373) Cluster #11 (MGG-11)

- $\sigma_r = 11.4 \pm 0.8 \mathrm{km/s}$
- \bullet half-light radius $1.2\pm0.17 {\rm pc}$
- ullet kinetic mass $3.5\pm0.7 imes10^5M_{\odot}$
- Age ~ 10Myrs.

Very short relaxation time (less than 10Myrs)

A possible scenario

- 1. Massive stars sink to the center through relaxation
- 2. Supermassive star forms through collision and merging
- 3. This star collapses to a BH

Simulation

Portegies Zwart et al., Nature, Apr 15, 2004



This scinario seems to work

GRAPE-6

Special-purpose computer for N-body simulations. 64 Tflops peak — World's fastest computer as of 2002.



Basic idea of GRAPE

Special-purpose hardware for force calculation General-purpose host for all other calculation



Time integration,Force calculationIO, etcFlexibilityFlexibilityHigh performance

GRAPE-6 Processor pipeline



Calculates gravitational force, its first time derivative and potential.

Evolution of GRAPE systems



Year

Next-Generation GRAPE — GRAPE-DR

- Planned peak speed: 2 Pflops
- New architecture wider application range than previous GRAPEs
- primarily to get funded
- No force pipeline. SIMD programmable processor
- Planned completion year: FY 2008 (early 2009)

Processor architecture



- Float Mult
- Float add/sub
- Integer ALU
- 32-word registers
- 256-word memory
- communication port

Chip structure



Result output port

Collection of small processors.

512 processors on one chip 500MHz clock

Peak speed of one chip: 0.5 Tflops (20 times faster than GRAPE-6).

Development status



Sample chip delivered May 2006

Chip layout

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- 32PEs in 16 groups
- 18mm by 18mm

Prototype board



2nd prototype board. (Designed by Toshi Fukushige) Difference from the 1st one:

- **PCI-Express x8 interface**
- **On-board DRAM**
- Designed to run real applications

Summary

- Gravitational Many-body problem is fun
- Recently, numerical integration of *N*-body systems have become useful tool for modelling observations.
- Special-purpose computers help.